Renormalizations in softly broken $N = 1$ theories: Slavnov-Taylor identities

2000 J. Phys. A: Math. Gen. 33 6399

(http://iopscience.iop.org/0305-4470/33/36/309)

J. Phys. A: Math. Gen. **33** (2000) 6399–6412. Printed in the UK PII: S0305-4470(00)12765-9

Renormalizations in softly broken $N = 1$ theories: Slavnov–Taylor identities

Igor Kondrashuk†
SISSA–ISAS and INFN, Sezione di Trieste, Via Beirut 2-4, I-34013, Trieste, Italy
E-mail: ikond@sissa.it

Received 17 March 2000

Abstract. Slavnov–Taylor identities have been applied to perform explicitly the renormalization procedure for the softly broken $N = 1$ SYM. The result is in accordance with the previous results obtained at the level of the supergraph technique.

1. Introduction

One of the ways to break supersymmetry is to introduce into the supersymmetric theory interactions with background superfields that are spacetime independent. The relation between the theory with softly broken supersymmetry and its rigid counterpart has been studied in [1–6]. The investigation has been performed for singular parts of the effective actions of softly broken and rigid theories. Since the only modification of the classical action from the rigid case to the softly broken case is a replacement of coupling constants of the rigid theory with background superfields, the relation is simple and can be reduced to substitutions of these superfields into renormalization constants of the rigid theory instead of the rigid-theory couplings [4,5]. Later, a relation between full correlators of softly broken and unbroken SUSY quantum mechanics has been found [7]. More recently, nonperturbative results for the terms of the effective action which correspond to the case when chiral derivatives do not act on background superfields have been derived [8].

The renormalization of the soft theory has been made on the basis of the supergraph technique in [5]. Here we perform the renormalization procedure for the softly broken theory using Slavnov–Taylor identities.

The notation used for the D4 supersymmetry and for the classical action S^{R} (R means 'rigid') of the theory without softly broken supersymmetry is given in the appendix. To give the possibility of comparing with the case of softly broken supersymmetry the renormalization procedure for the rigid $N = 1$ SYM is reviewed in the appendix.

2. $N = 1$ softly broken theories

The classical action S^{S} (the superscript S means 'soft') with softly broken supersymmetry repeats the rigid action S^{R} (A2) except for the replacement couplings of the theory with

† On leave of absence from LNP, JINR, Dubna, Russia.

background x-independent superfields,

$$S^{\rm S} = \int d^4y\, d^2\theta S \frac{1}{2^7} \,{\rm Tr}\, W_\alpha W^\alpha + \int d^4\bar{y}\, d^2\bar{\theta} \bar{S} \frac{1}{2^7} \,{\rm Tr}\, \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}}$$
$$+ \int d^4x\, d^2\theta\, d^2\bar{\theta}\, \bar{\Phi}^i ({\rm e}^V)_i{}^j K_j{}^k \Phi_k + \int d^4y\, d^2\theta [\tilde{y}^{ijk} \Phi_i \Phi_j \Phi_k + \tilde{M}^{ij} \Phi_i \Phi_j]$$
$$+ \int d^4\bar{y}\, d^2\bar{\theta} [\bar{\tilde{y}}_{ijk} \bar{\Phi}^i \bar{\Phi}^j \bar{\Phi}^k + \overline{\tilde{M}}_{ij} \bar{\Phi}^i \bar{\Phi}^j]. \tag{1}$$

The indices of the matter superfields are reducible. They run over irreducible representations and members of them. The external background x-independent superfields S, K_i^j and $\tilde{y}_{ijk}$ are

$$S = \frac{1}{g^2}(1 - 2m_A\theta^2) \qquad \bar{S} = \frac{1}{g^2}(1 - 2\bar{m}_A\bar{\theta}^2)$$
$$K_i{}^j = \delta_i^j + (m^2)_i^j \theta^2 \bar{\theta}^2$$
$$\tilde{y}_{ijk} = y_{ijk} + A_{ijk}\theta^2 \qquad \bar{\tilde{y}}_{ijk} = \bar{y}_{ijk} + \bar{A}_{ijk}\bar{\theta}^2$$
$$\tilde{M}_{ij} = M_{ij} + B_{ij}\theta^2 \qquad \overline{\tilde{M}}_{ij} = \bar{M}_{ij} + \bar{B}_{ij}\bar{\theta}^2.$$

These superfields break supersymmetry in a soft way since they are not included in the supersymmetry transformation at the component level.

3. Slavnov–Taylor identities

In the rest of the paper we concentrate on the gauge part of the action. The renormalization of the chiral matter superfields is trivial and is evident from the supergraph technique [1, 5].

To fix the gauge we have to add the gauge fixing term and the ghost terms to the action (1), which we choose in a slightly different manner in comparison with the rigid case (A3),

$$\int d^4x\, d^2\theta\, d^2\bar{\theta} \frac{1}{16} \,{\rm Tr} \left(\bar{D}^2 \frac{V}{\sqrt{\tilde{\alpha}}}\right) \left(D^2 \frac{V}{\sqrt{\tilde{\alpha}}}\right)$$
$$+ \int d^4y\, d^2\theta\, \frac{\rm i}{2} \,{\rm Tr}\, b\bar{D}^2 \left(\frac{\delta_{\bar{c},c} V}{\sqrt{\tilde{\alpha}}}\right) + \int d^4\bar{y}\, d^2\bar{\theta}\, \frac{\rm i}{2} \,{\rm Tr}\; \bar{b} D^2 \left(\frac{\delta_{\bar{c},c} V}{\sqrt{\bar{\tilde{\alpha}}}}\right)$$

where b and $\bar{b}$ are antighost chiral and antichiral superfields, and c and $\bar{c}$ are ghost chiral and antichiral superfields, respectively. Throughout this paper we consider the non-zero highest components of the couplings as an insertion into the rigid theory supergraphs. Such a choice of the gauge fixing term and the ghost terms means that we fix the gauge arbitrariness by imposing the condition

$$D^2 \frac{V(x, \theta, \bar{\theta})}{\sqrt{\tilde{\alpha}}} = \bar{f}(\bar{y}, \bar{\theta}) \qquad \bar{D}^2 \frac{V(x, \theta, \bar{\theta})}{\sqrt{\bar{\tilde{\alpha}}}} = f(y, \theta) \tag{2}$$

where f and $\bar{f}$ are arbitrary chiral and antichiral functions. This allows us to consider the gauge fixing constant $\tilde{\alpha}$ as an external x-independent background superfield on the same foot with the soft couplings and the soft masses of the softly broken action (1). This modification of the gauge fixing condition is important even at the level of supergraph technique [5]. As it will be clear below this modification is the necessary way to remove divergences from the effective action of the softly broken theory using Slavnov–Taylor identities.

Hence, the total gauge part of the classical action (1) is

$$S^{\rm S}_{\rm gauge} = \int d^4y\, d^2\theta S \frac{1}{2^7} \,{\rm Tr}\, W_\alpha W^\alpha + \int d^4\bar{y}\, d^2\bar{\theta}\, \bar{S} \frac{1}{2^7} \,{\rm Tr}\, \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}}$$

$$+\int \mathrm{d}^4x\,\mathrm{d}^2\theta\,\mathrm{d}^2\bar\theta\frac{1}{16}\,\mathrm{Tr}\left(\bar D^2\frac{V}{\sqrt{\tilde\alpha}}\right)\left(D^2\frac{V}{\sqrt{\tilde\alpha}}\right)$$
$$+\int \mathrm{d}^4y\,\mathrm{d}^2\theta\frac{\mathrm{i}}{2}\,\mathrm{Tr}\,b\bar D^2\left(\frac{\delta_{\bar c,c}V}{\sqrt{\tilde\alpha}}\right)+\int \mathrm{d}^4\bar y\,\mathrm{d}^2\bar\theta\,\frac{\mathrm{i}}{2}\,\mathrm{Tr}\,\bar b\;D^2\left(\frac{\delta_{\bar c,c}V}{\sqrt{\tilde\alpha}}\right). \qquad (3)$$

The action (3) is invariant under the same BRST symmetry as the rigid gauge action (3) except for the transformation of the antighost superfields, which is a little different from what we have in the rigid case (A7):

$$\begin{aligned} &\mathrm{e}^V \to \mathrm{e}^{\mathrm{i}\bar c\varepsilon}\mathrm{e}^V\mathrm{e}^{\mathrm{i}c\varepsilon} \qquad && \delta b = \frac{1}{32}\left(\bar D^2D^2\frac{V}{\sqrt{\tilde\alpha}}\right)\varepsilon \\ &c \to c + \mathrm{i}c^2\varepsilon && \delta\bar b = \frac{1}{32}\left(D^2\bar D^2\frac{V}{\sqrt{\tilde\alpha}}\right)\varepsilon \\ &\bar c \to \bar c - \mathrm{i}\bar c^2\varepsilon && \end{aligned} \qquad (4)$$

with a Hermitian–Grassmannian parameter ε, $\varepsilon^\dagger = \varepsilon$.

The path integral describing the quantum soft theory is defined in the same way as the path integral (A8) of the rigid theory,

$$Z[J,\eta,\bar\eta,\rho,\bar\rho,K,L,\bar L] = \int \mathrm{d}V\,\mathrm{d}c\,\mathrm{d}\bar c\,\mathrm{d}b\,\mathrm{d}\bar b\exp\mathrm{i}[S^{\mathrm{S}}_{\mathrm{gauge}}$$
$$+2\,\mathrm{Tr}(JV + \mathrm{i}\eta c + \mathrm{i}\bar\eta\bar c + \mathrm{i}\rho b + \mathrm{i}\bar\rho\bar b) + 2\,\mathrm{Tr}(\mathrm{i}K\delta_{\bar c,c}V + Lc^2 + \bar L\bar c^2)]. \qquad (5)$$

The third term in the brackets is the BRST invariant since the external superfields K and L are BRST invariant by definition. All fields in the path integral are in the adjoint representation of the gauge group. For the sake of brevity we omit the symbol of integration in the terms with external sources, keeping in mind that it is the full superspace measure for vector superfields and the chiral measure for chiral superfields.

The ghost equation that is a reflection of invariance of the path integral (5) under the change of variables

$$b \to b + \varepsilon \qquad \bar b \to \bar b + \bar\varepsilon$$

with an arbitrary chiral superfield ε must be modified in comparison with the ghost equation of the rigid theory (A9) taking into account the modified BRST transformation of the antighost field (4). As the result, two ghost equations can be derived

$$\bar\rho - \mathrm{i}\frac{1}{4}D^2\frac{1}{\sqrt{\tilde\alpha}}\frac{\delta W}{\delta K} = 0 \qquad \rho - \mathrm{i}\frac{1}{4}\bar D^2\frac{1}{\sqrt{\tilde\alpha}}\frac{\delta W}{\delta K} = 0.$$

The Legendre transformation (A11) that has been made in the appendix for the rigid case can be repeated here without changes. Taking into account the relations (A10) and (A12), the ghost equations can be represented as

$$\frac{\delta\Gamma}{\delta\bar b} - \frac{1}{4}D^2\frac{1}{\sqrt{\tilde\alpha}}\frac{\delta\Gamma}{\delta K} = 0 \qquad \frac{\delta\Gamma}{\delta b} - \frac{1}{4}\bar D^2\frac{1}{\sqrt{\tilde\alpha}}\frac{\delta\Gamma}{\delta K} = 0. \qquad (6)$$

If the change of fields (4) in the path integral (5) is made we obtain the Slavnov–Taylor identity as the result of invariance of the integral (5) under a change of variables. There is complete analogy with the rigid case (A14) except for a little difference caused by the modified transformation of the antighost superfield in (4). The Slavnov–Taylor identities for the theory (5) are

$$\mathrm{Tr}\left[\frac{\delta\Gamma}{\delta V}\frac{\delta\Gamma}{\delta K} - \mathrm{i}\frac{\delta\Gamma}{\delta c}\frac{\delta\Gamma}{\delta L} + \mathrm{i}\frac{\delta\Gamma}{\delta\bar c}\frac{\delta\Gamma}{\delta\bar L} - \frac{\delta\Gamma}{\delta b}\left(\frac{1}{32}\bar D^2D^2\frac{V}{\sqrt{\tilde\alpha}}\right) - \frac{\delta\Gamma}{\delta\bar b}\left(\frac{1}{32}D^2\bar D^2\frac{V}{\sqrt{\tilde\alpha}}\right)\right] = 0. \qquad (7)$$

4. Renormalizations of the softly broken SYM

The identities (6) and (7) allow us to remove all possible divergences from the effective action Γ by rescaling superfields and couplings in the classical action (3). Indeed, the identity (6) restricts the dependence of Γ on the antighost superfields and on the external source K to an arbitrary dependence on their combination

$$(b+\bar{b})\frac{1}{\sqrt{\tilde{\alpha}}}+K.$$

This means that the corresponding singular part of the effective action is

$$\int \mathrm{d}^4x\,\mathrm{d}^2\theta\,\mathrm{d}^2\bar{\theta}2\mathrm{i}\,\mathrm{Tr}\left((b+\bar{b})\frac{1}{\sqrt{\tilde{\alpha}}}+K\right)\tilde{A}(x,\theta,\bar{\theta})$$

where $\tilde{A}(x,\theta,\bar{\theta})$ is a combination of $c, \bar{c}, V$. By index counting arguments we know that the singular part repeats the structure of the classical action (3) up to coefficients. Hence, $\tilde{A}(x,\theta,\bar{\theta})$ starts from the $\tilde{z}_1(c+\bar{c})$, since Γ is Hermitian. Here $\tilde{z}_1$ is a constant that can be found by using the supergraph technique.

Now we can compare the renormalization constants $\tilde{z}_1$ and z_1. The constant z_1 is obtained from $\tilde{z}_1$ by putting all higher components of the soft couplings, of the soft masses, and of the gauge fixing coupling $\tilde{\alpha}$ in the action (1) equal to zero. In this case z_1 is a little different constant than that is appeared in the appendix, since that rigid theory (A8) has another gauge fixing condition. Taking into account arguments based on the index of divergence and keeping in mind the absence of chiral derivatives in the ghost parts of the actions (A6) and (3) we can see that

$$\begin{aligned}\tilde{z}_1(\tilde{g}^2,\sqrt{\tilde{\alpha}}) &= z_1(g^2\to\tilde{g}^2,\sqrt{\alpha}\to\sqrt{\tilde{\alpha}})\\ \tilde{g}^2 &= g^2(1+m_A\theta^2+\bar{m}_A\bar{\theta}^2+2m_A\bar{m}_A\theta^2\bar{\theta}^2)=\left(\frac{S+\bar{S}}{2}\right)^{-1}.\end{aligned} \tag{8}$$

The substitution $g^2 \to \tilde{g}^2$ becomes obvious if we remember that we consider higher components of the gauge coupling as insertions into the vector propagator and into the vector vertices in supergraphs [1, 5]. In short, the arguments of [1, 5] are the following. Since the action of a chiral derivative on spurions means decreasing the index of divergence inherited from a rigid diagram, a supergraph with logarithmic divergence becomes convergent in this case. Hence, for the divergent part all spurions must be taken out of a supergraph together with rigid couplings.

For the same reason we take out of a supergraph the external superfield $\sqrt{\tilde{\alpha}}$. Under the condition

$$\tilde{\alpha}=\tilde{g}^2$$

we obtain the result obtained in the [5] at the supergraph level for the renormalization constants that become x-independent vector superfields,

$$\tilde{z}_1=z_1(g^2\to\tilde{g}^2).$$

In the same way as in the rigid case, the Slavnov–Taylor identity (7) fixes the coefficient before the longitudinal part of the two-point vector Green function. Indeed, by using projectors from (A1) the infinite part of the two-point vector correlator can be decomposed as

$$\begin{aligned}V(D,\tilde{z}_a,\bar{D},\tilde{z}_b,D,\tilde{z}_c,\bar{D},\tilde{z}_d)V &= V(D,\tilde{z}_a,\bar{D},\tilde{z}_b,D,\tilde{z}_c,\bar{D},\tilde{z}_d)\frac{D^\alpha\bar{D}^2D_\alpha}{8\Box}V\\ &\quad -V(D,\tilde{z}_a,\bar{D},\tilde{z}_b,D,\tilde{z}_c,\bar{D},\tilde{z}_d)\frac{D^2\bar{D}^2+\bar{D}^2D^2}{16\Box}V\end{aligned} \tag{9}$$

where the four derivatives in parenthesis can stand in some (in general, unknown) way. The difference from the analogous rigid case decomposition of the two-point vector correlator is in a possible presence of x-independent background superfields $\tilde{z}_a, \tilde{z}_b, \tilde{z}_c, \tilde{z}_d$ between these derivatives.

The identity (7) means that these four derivatives in the second term of this decomposition must cancel $\Box$ in the denominator and the longitudinal term is reduced to the form

$$\tilde{z}_2 \frac{1}{32} \frac{V}{\sqrt{\tilde{\alpha}}} (D^2 \bar{D}^2 + \bar{D}^2 D^2) \tilde{z}_2 \frac{V}{\sqrt{\tilde{\alpha}}}.$$

It is not difficult to check that the Slavnov–Taylor identity also gives that $\tilde{z}_2 = 1$, that is, there is no infinite correction to the longitudinal part of the two-point vector Green function in the soft case. The same arguments can be applied even in the case of the total effective action, taking into account the whole dependence of the effective action Γ on the combination

$$(b + \bar{b}) \frac{1}{\sqrt{\tilde{\alpha}}} + K.$$

Hence, there is no finite correction to the longitudinal part of the two-point vector correlator in the soft case.

Now it is necessary to consider contributions in $\tilde{A}(x, \theta, \bar{\theta})$ of the next orders in fields. For example, the third-order terms can be presented as

$$\int \mathrm{d}^4x \, \mathrm{d}^2\theta \, \mathrm{d}^2\bar{\theta} 2\mathrm{i} \, \mathrm{Tr} \left((b + \bar{b}) \frac{1}{\sqrt{\tilde{\alpha}}} + K \right) [\tilde{z}_1 (c + \bar{c}) + \tilde{z}_4 (Vc + \bar{c}V) + \tilde{z}_5 (cV + V\bar{c})]$$
$$+ \int \mathrm{d}^4y \, \mathrm{d}^2\theta \, 2 \, \mathrm{Tr} \, \tilde{z}_6 L c^2 + \int \mathrm{d}^4\bar{y} \, \mathrm{d}^2\bar{\theta} \, 2 \, \mathrm{Tr} \, \bar{\tilde{z}}_6 \bar{L} \bar{c}^2. \tag{10}$$

By the no-renormalization theorem for the superpotential [9] we obtain

$$\tilde{z}_6 = \bar{\tilde{z}}_6 = 1.$$

To fix the constants $\tilde{z}_4$ and $\tilde{z}_5$, we make the change of variables in the effective action Γ

$$\Gamma[V, c, \bar{c}, b, \bar{b}, K, L, \bar{L}] = \Gamma[V(\tilde{V}), c, \bar{c}, b, \bar{b}, K(\tilde{K}), L, \bar{L}] = \tilde{\Gamma}[\tilde{V}, c, \bar{c}, b, \bar{b}, \tilde{K}, L, \bar{L}]$$
$$V = \tilde{V} \tilde{z}_1 \qquad K = \frac{\tilde{K}}{\tilde{z}_1}. \tag{11}$$

The Slavnov–Taylor identity (7) in the new variables is

$$\mathrm{Tr} \left[\frac{\delta\tilde{\Gamma}}{\delta\tilde{V}} \frac{\delta\tilde{\Gamma}}{\delta\tilde{K}} - \mathrm{i} \frac{\delta\tilde{\Gamma}}{\delta c} \frac{\delta\tilde{\Gamma}}{\delta L} + \mathrm{i} \frac{\delta\tilde{\Gamma}}{\delta\bar{c}} \frac{\delta\tilde{\Gamma}}{\delta\bar{L}} - \frac{\delta\tilde{\Gamma}}{\delta b} \left(\frac{1}{32} \bar{D}^2 D^2 \frac{\tilde{V}\tilde{z}_1}{\sqrt{\tilde{\alpha}}} \right) - \frac{\delta\tilde{\Gamma}}{\delta\bar{b}} \left(\frac{1}{32} D^2 \bar{D}^2 \frac{\tilde{V}\tilde{z}_1}{\sqrt{\tilde{\alpha}}} \right) \right] = 0. \tag{12}$$

The part of the effective action (10) in the new variables looks like

$$\int \mathrm{d}^4x \, \mathrm{d}^2\theta \, \mathrm{d}^2\bar{\theta} \, 2\mathrm{i} \, \mathrm{Tr} \left((b + \bar{b}) \frac{\tilde{z}_1}{\sqrt{\tilde{\alpha}}} + \tilde{K} \right) [(c + \bar{c}) + \tilde{z}_4' (\tilde{V}c + \bar{c}\tilde{V}) + \tilde{z}_5' (c\tilde{V} + \tilde{V}\bar{c})]$$
$$+ \int \mathrm{d}^4y \, \mathrm{d}^2\theta \, 2 \, \mathrm{Tr} \, L c^2 + \int \mathrm{d}^4\bar{y} \, \mathrm{d}^2\bar{\theta} \, 2 \, \mathrm{Tr} \, \bar{L} \bar{c}^2 \tag{13}$$

where $\tilde{z}_4'$ and $\tilde{z}_5'$ are new constants.

The higher-order terms in the brackets of (13) are restored unambiguously by themselves in the iterative way due to the first three terms in the modified identities (12). As the result, we have

$$\int \mathrm{d}^4x \, \mathrm{d}^2\theta \, \mathrm{d}^2\bar{\theta} \, 2\mathrm{i} \, \mathrm{Tr} \left((b + \bar{b}) \frac{\tilde{z}_1}{\sqrt{\tilde{\alpha}}} + \tilde{K} \right) [\delta_{\bar{c},c} \tilde{V}]. \tag{14}$$

Now it is necessary to consider the transversal part of the two-point vector correlator. Having made the change of variables in the effective action (11), we see that the only structures of derivatives in the two-point vector Green function

$$\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \tilde{z}_1 \tilde{V}(D, \tilde{z}_a, \bar{D}, \tilde{z}_b, D, \tilde{z}_c, \bar{D}, \tilde{z}_d)\tilde{z}_1\tilde{V}$$

which are allowed by the modified identities (12) are

$$\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, S\frac{1}{2^5} f(S)(D_\alpha \tilde{V})(\bar{D}^2 D^\alpha \tilde{V}) + \mathrm{H.c.}$$
$$+ \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \mathrm{Tr}\, \frac{1}{32}\frac{\tilde{z}_1\tilde{V}}{\sqrt{\tilde{\alpha}}}(D^2\bar{D}^2 + \bar{D}^2 D^2)\frac{\tilde{z}_1\tilde{V}}{\sqrt{\tilde{\alpha}}}. \tag{15}$$

Here we have used the dependence of the singular part of $\tilde{\Gamma}$ on the external source $\tilde{K}$ which has already been fixed by (14). The function f must be a chiral superfield.

Since the function f is obtained from the background superfields in the case when chiral derivatives do not act on them, it can be obtained as the result of the change of rigid theory couplings with background superfields. But we have only one chiral background superfield which is the soft gauge coupling S. Hence, $f(S)$ can be obtained from the corresponding coefficient of the rigid theory by the change

$$\frac{1}{g^2} \to S.$$

In the limit of constant gauge coupling we have

$$\int \mathrm{d}^4y\, \mathrm{d}^2\theta \frac{1}{g^2}\frac{1}{2^7} z_1^2\, z_3 (\bar{D}^2 D_\alpha \tilde{V})(\bar{D}^2 D^\alpha \tilde{V}) + \mathrm{H.c.}$$

where z_1 and z_3 are renormalization constants of the rigid theory. Hence, we can derive that

$$f(S)|_{\theta^2=0} = z_3\, z_1^2 = z_{g^2} \qquad f(S) \equiv \tilde{z}_S(S) = z_{g^2}\left(\frac{1}{g^2} \to S\right). \tag{16}$$

Hence, the renormalization constants $(\tilde{z}_S, z_{g^2})$ are not related as in the rule (8) for the pair $(\tilde{z}_1, z_1)$, but are related in the holomorphic way (16).

The first term in the modified identity (12) will restore in the iterative way higher-order terms starting from the bilinear transversal two-point correlator (15). Hence, the result of this restoration is

$$\int \mathrm{d}^4y\, \mathrm{d}^2\theta\, S\frac{1}{2^7}\tilde{z}_S\, \mathrm{Tr}\, W_\alpha(\tilde{V}) W^\alpha(\tilde{V}) + \mathrm{H.c.}. \tag{17}$$

Hence, chiral (or antichiral) parts of the vector renormalization couplings are of importance only if we talk about the renormalization of the soft gauge coupling S. This result is in accordance with our previous results [5] obtained from the analysis of divergences in supergraphs.

The following notation is used for brevity in (17):

$$W^\alpha(V) \equiv \bar{D}^2(\mathrm{e}^{-V} D^\alpha \mathrm{e}^V).$$

The singular part of the effective action $\tilde{\Gamma}$ can be written as a combination of (17) and (14),

$$\tilde{\Gamma}_{\mathrm{sing}} = \int \mathrm{d}^4y\, \mathrm{d}^2\theta\, S\frac{1}{2^7}\tilde{z}_S\, \mathrm{Tr}\, W_\alpha(\tilde{V}) W^\alpha(\tilde{V}) + \mathrm{H.c.}$$
$$+ \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \mathrm{Tr}\, \frac{1}{32}\frac{\tilde{z}_1\tilde{V}}{\sqrt{\tilde{\alpha}}}(D^2\bar{D}^2 + \bar{D}^2 D^2)\frac{\tilde{z}_1\tilde{V}}{\sqrt{\tilde{\alpha}}}$$
$$+ \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, 2\mathrm{i}\, \mathrm{Tr}\left((b+\bar{b})\frac{\tilde{z}_1}{\sqrt{\tilde{\alpha}}} + \tilde{K}\right)[\delta_{\bar{c},c}\tilde{V}]. \tag{18}$$

Now we should go back to the initial variables V and K, that is, we should make the change of variables in $\tilde{\Gamma}$ opposite to (11). Hence, the singular part of the effective action which corresponds to the theory with the classical action (3) is

$$\Gamma_{\text{sing}} = \int \mathrm{d}^4y\, \mathrm{d}^2\theta\, S \frac{1}{2^7} \tilde{z}_S \operatorname{Tr} W_\alpha \left(\frac{V}{\tilde{z}_1}\right) W^\alpha \left(\frac{V}{\tilde{z}_1}\right) + \text{H.c.}$$
$$+ \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \operatorname{Tr} \frac{1}{32} \frac{V}{\sqrt{\tilde{\alpha}}} (D^2\bar{D}^2 + \bar{D}^2D^2) \frac{V}{\sqrt{\tilde{\alpha}}}$$
$$+ \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, 2\mathrm{i} \operatorname{Tr} \left((b + \bar{b}) \frac{\tilde{z}_1}{\sqrt{\tilde{\alpha}}} + K \tilde{z}_1 \right) \left[\delta_{\bar{c},c} \left(\frac{V}{\tilde{z}_1} \right) \right]. \tag{19}$$

Hence, all divergences can be removed from Γ_{sing} by the following rescaling of fields and couplings in the path integral (5):

$$V = V_R\, \tilde{z}_1 \qquad S = S_R\, \tilde{z}_S^{-1} \qquad \sqrt{\tilde{\alpha}} = \tilde{z}_1 \sqrt{\tilde{\alpha}_R} \qquad K = K_R\, \tilde{z}_1^{-1}. \tag{20}$$

5. Conclusions and discussion

In this paper the relations (8) and (16) between the renormalization constants of the softly broken SYM and their prototypes from the corresponding rigid theory which have been found in [4] starting from the Hisano–Shifman nonperturbative result [2] and in [5] starting from the supergraph technique for vector vertices have been derived from the Slavnov–Taylor identities. It has been shown that the modification (2) of the gauge fixing condition is necessary and important for the renormalization procedure in the softly broken SYM.

It is clear from the analysis performed here that instead of a spacetime-independent soft gauge coupling we could consider any chiral superfield without changing the proof given in this paper. This may be important for the models in which supersymmetry breaking is communicated to the observable world through interactions with messengers. In these models S is a messenger superfield which can gain the vacuum expectation value for its highest component due to interactions with a hidden sector [3, 6, 13]. This idea with a toy model for a hidden sector has been considered in [14].

As to the relation between chiral matter renormalization constants of the soft theory and those of the rigid theory, it has been established in [1] as substitutions of background superfields into rigid renormalization constants instead of rigid couplings. The result of these substitutions can be described as in [4, 5] through differential operators that act in the coupling constant space of the rigid theory. The same operators can be used to relate soft and rigid renormalization group functions [4, 5]. Possible applications of the relations between soft and rigid RG functions to the analysis of phenomenological models can be found in [3, 5, 6, 15].

All the derivations proposed in this paper make sense only if we have fixed a gauge invariant regularization and defined a renormalization scheme to remove the infinities. In this work we implied the DRED scheme [16], that is the only practical regulator in order to be able to calculate higher-order effects in any supersymmetric theory including MSSM. In this case Slavnov–Taylor identities (12) do not forbid a new gauge invariant term [17] of the effective action $\tilde{\Gamma}$ (18)

$$\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, g_{mn}^{(\epsilon)} \operatorname{Tr} \Gamma_m \Gamma_n$$

where $g_{mn}^{(\epsilon)}$ is the metric in the 2ϵ compactified dimensions and Γ_m is the superfield gauge connection defined by

$$\Gamma_m = \tfrac{1}{2} \sigma_m{}^{\alpha\dot{\beta}} \bar{D}_{\dot{\beta}} (\mathrm{e}^{-\tilde{V}} D_\alpha \mathrm{e}^{\tilde{V}}).$$

This term generates so-called ϵ scalar masses in the course of the renormalization procedure. Indeed, one can see that at the component level the $\theta^2\bar{\theta}^2$ component of $\tilde{z}_1$ produces ϵ scalar masses [18] when we are replacing $\tilde{V}$ with $\frac{V}{\tilde{z}_1}$ in $\tilde{\Gamma}$ to obtain the singular part of the effective action $\Gamma_{\rm sing}$ (19). Even if initially the ϵ scalar masses are equal to zero, this condition is unstable under renormalizations and, hence, such a counterterm must be added. As has been found in [18] the ϵ scalar mass dependence of the two-loop β functions can be completely removed by a slight modification of the DRED scheme to the $\overline{\rm DR}'$ scheme. The way to generalize this scheme to all orders of the perturbation theory has been proposed in [6]. However, based on the explicit presence of this contribution at the two-loop level in the component formalism [18], it has been stated in [19] that the contribution of the ϵ scalar mass renormalization should be taken into account in the physical soft scalar mass β functions. It is possible to determine this contribution at all orders of the perturbation theory by requiring the existence of a set of renormalization group invariant relations between soft couplings and masses as has been done in [19] for the $\overline{\rm DR}'$ scheme and further developed in [20, 21] to other schemes.

Acknowledgments

I am grateful to Antonio Masiero for many discussions. This work is supported by INFN.

Appendix

Our supersymmetric notations are

$$
\begin{array}{ll}
(\psi\sigma_m\bar{\chi}) \equiv \psi_\alpha {\sigma_m}^{\alpha\dot{\beta}}\bar{\chi}_{\dot{\beta}} & (\psi\sigma_m\bar{\chi})^\dagger = (\chi\sigma_m\bar{\psi}) \\
{\sigma_m}^{\alpha\dot{\beta}} = (\mathrm{I}, \sigma_i) & \bar{\sigma}_m^{\dot{\beta}\alpha} = {\sigma_m}^{\alpha\dot{\beta}} \\
\chi^\alpha = \epsilon^{\alpha\beta}\chi_\beta & \epsilon^{12} = -1 \\
\theta^2 = -\theta_\alpha\theta^\alpha & \bar{\theta}^2 = -\bar{\theta}^{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}} \Rightarrow {\theta^2}^\dagger = \bar{\theta}^2 \\
\partial^\alpha\theta_\beta = \delta^\alpha_\beta \Rightarrow \bar{\theta}_{\dot{\beta}}\overleftarrow{\bar{\partial}^{\dot{\alpha}}} = \delta^{\dot{\alpha}}_{\dot{\beta}} & \\
\int \mathrm{d}^2\theta\,\theta^2 \equiv \frac{1}{4}\partial^2\theta^2 = -\frac{1}{4}\partial_\alpha\partial^\alpha\theta^2 = -1 & \int \mathrm{d}^2\bar{\theta}\,\bar{\theta}^2 \equiv \frac{1}{4}\overleftarrow{\bar{\partial}^2}\bar{\theta}^2 = -\overleftarrow{\bar{\partial}^{\dot{\alpha}}}\,\overleftarrow{\bar{\partial}_{\dot{\alpha}}}\bar{\theta}^2 = -1.
\end{array}
$$

The algebra of supersymmetry and covariant derivatives is

$$
\begin{array}{ll}
\multicolumn{2}{l}{\varepsilon_\alpha Q^\alpha + \bar{Q}^{\dot{\alpha}}\bar{\varepsilon}_{\dot{\alpha}} = \varepsilon_\alpha(\partial^\alpha + \mathrm{i}{\sigma_m}^{\alpha\dot{\beta}}\bar{\theta}_{\dot{\beta}}\partial_m) + (\overleftarrow{\bar{\partial}^{\dot{\alpha}}} - \mathrm{i}\theta_\beta{\sigma_m}^{\beta\dot{\alpha}}\partial_m)\bar{\varepsilon}_{\dot{\alpha}}} \\
Q^\alpha = \partial^\alpha + \mathrm{i}{\sigma_m}^{\alpha\dot{\beta}}\bar{\theta}_{\dot{\beta}}\partial_m & \bar{Q}^{\dot{\alpha}} = \overleftarrow{\bar{\partial}^{\dot{\alpha}}} - \mathrm{i}\theta_\beta{\sigma_m}^{\beta\dot{\alpha}}\partial_m \\
\{Q^\alpha, \bar{Q}^{\dot{\beta}}\} = -2\mathrm{i}{\sigma_m}^{\alpha\dot{\beta}}\partial_m & \{Q^\alpha, Q^\beta\} = \{\bar{Q}^{\dot{\alpha}}, \bar{Q}^{\dot{\beta}}\} = 0 \\
\{D^\alpha, \bar{Q}^{\dot{\beta}}\} = 0 & \\
D^\alpha = \partial^\alpha - \mathrm{i}(\sigma_m\bar{\theta})^\alpha\partial_m & \bar{D}^{\dot{\alpha}} = \overleftarrow{\bar{\partial}^{\dot{\alpha}}} + \mathrm{i}(\theta\sigma_m)^{\dot{\alpha}}\partial_m \\
\{D^\alpha, \bar{D}^{\dot{\beta}}\} = 2\mathrm{i}{\sigma_m}^{\alpha\dot{\beta}}\partial_m & \{D^\alpha, D^\beta\} = \{\bar{D}^{\dot{\alpha}}, \bar{D}^{\dot{\beta}}\} = 0 \\
(D^\alpha\bar{D}^2D_\alpha)^\dagger = D^\alpha\bar{D}^2D_\alpha & \\
\multicolumn{2}{l}{\dfrac{D^\alpha\bar{D}^2D_\alpha}{8\Box} - \dfrac{D^2\bar{D}^2 + \bar{D}^2D^2}{16\Box} = 1} \\
\multicolumn{2}{l}{\Box = \eta_{mn}\partial_m\partial_n = \dfrac{\partial}{\partial x^0}\dfrac{\partial}{\partial x^0} - \dfrac{\partial}{\partial x^1}\dfrac{\partial}{\partial x^1} - \cdots} \\
\eta_{mn} = (1, -1, -1, -1). &
\end{array}
\tag{A1}
$$

The classical rigid action S^{R} of the supersymmetric theory with $N = 1$ supersymmetry without soft terms in the superfield formalism is

$$\int \mathrm{d}^4y\, \mathrm{d}^2\theta \frac{1}{g^2}\frac{1}{2^7}\,\mathrm{Tr}\, W_\alpha W^\alpha + \int \mathrm{d}^4\bar{y}\, \mathrm{d}^2\bar{\theta} \frac{1}{g^2}\frac{1}{2^7}\,\mathrm{Tr}\, \bar{W}^{\dot\alpha}\bar{W}_{\dot\alpha} + \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \bar{\Phi}^i (\mathrm{e}^V)_i{}^j \Phi_j$$
$$+ \int \mathrm{d}^4y\, \mathrm{d}^2\theta\, [y^{ijk}\Phi_i\Phi_j\Phi_k + M^{ij}\Phi_i\Phi_j] + \int \mathrm{d}^4\bar{y}\, \mathrm{d}^2\bar{\theta}\, [\bar{y}_{ijk}\bar{\Phi}^i\bar{\Phi}^j\bar{\Phi}^k + \bar{M}_{ij}\bar{\Phi}^i\bar{\Phi}^j]. \tag{A2}$$

Here W_α is the supertensity,

$$W^\alpha \equiv \bar{D}^2(\mathrm{e}^{-V} D^\alpha \mathrm{e}^V)$$

V is a real superfield, $V^\dagger = V$. All fields of the real supermultiplet are in the adjoint representation of the gauge group

$$W_\alpha = W_\alpha^a T^a \qquad \mathrm{Tr}(T^a T^b) = \tfrac{1}{2}\delta^{ab} \qquad (T^a)^\dagger = T^a.$$

To fix the gauge we have to add the gauge fixing term and the ghost terms to the action (A2), which can be chosen in the standard form [9]

$$\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \frac{1}{16}\frac{1}{\alpha}\,\mathrm{Tr}(\bar{D}^2 V)(D^2 V)$$
$$+ \int \mathrm{d}^4y\, \mathrm{d}^2\theta\, \frac{\mathrm{i}}{2}\,\mathrm{Tr}\, b\bar{D}^2\delta_{\bar{c},c}V + \int \mathrm{d}^4\bar{y}\, \mathrm{d}^2\bar{\theta}\, \frac{\mathrm{i}}{2}\,\mathrm{Tr}\, \bar{b}D^2\delta_{\bar{c},c}V. \tag{A3}$$

where b and $\bar{b}$ are the antighost chiral and antichiral superfields, and c and $\bar{c}$ are the ghost chiral and antichiral superfields. Such a choice of the gauge fixing and the ghost terms means that we fix the gauge arbitrariness by imposing the condition

$$D^2V(x,\theta,\bar{\theta}) = \bar{f}(\bar{y},\bar{\theta}) \qquad \bar{D}^2V(x,\theta,\bar{\theta}) = f(y,\theta)$$

where $\bar{f}$ and f are arbitrary antichiral and chiral functions. Under the gauge transformation the vector superfield V transforms as

$$\mathrm{e}^V \to \mathrm{e}^{\bar{\Lambda}}\mathrm{e}^V\mathrm{e}^\Lambda \tag{A4}$$

where $\bar{\Lambda}$, Λ are antichiral and chiral degrees of gauge freedom. We define $\delta_{\bar{\Lambda},\Lambda}V$ as the solution to the equation

$$\mathrm{e}^{V+\delta_{\bar{\Lambda},\Lambda}V} = \mathrm{e}^{\bar{\Lambda}}\mathrm{e}^V\mathrm{e}^\Lambda$$

with infinitesimal fields $\bar{\Lambda}$, Λ. This equation can be transformed to the form

$$\mathrm{e}^V(\delta_{\bar{\Lambda},\Lambda}V) - (\delta_{\bar{\Lambda},\Lambda}V)\mathrm{e}^V = [V,\bar{\Lambda}]\mathrm{e}^V + \mathrm{e}^V[V,\Lambda] \tag{A5}$$

that can be solved [9] as

$$\delta_{\bar{\Lambda},\Lambda}V = \frac{V}{2}\coth\frac{V}{2} \wedge (\bar{\Lambda}+\Lambda) - \frac{V}{2} \wedge (\bar{\Lambda}-\Lambda).$$

Hence, the total gauge part of the classical action (A2) is

$$S^{\mathrm{R}}_{\mathrm{gauge}} = \int \mathrm{d}^4y\, \mathrm{d}^2\theta\, \frac{1}{g^2}\frac{1}{2^7}\,\mathrm{Tr}\, W_\alpha W^\alpha + \int \mathrm{d}^4\bar{y}\, \mathrm{d}^2\bar{\theta}\, \frac{1}{g^2}\frac{1}{2^7}\,\mathrm{Tr}\, \bar{W}^{\dot\alpha}\bar{W}_{\dot\alpha}$$
$$+ \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta} \frac{1}{16}\frac{1}{\alpha}\,\mathrm{Tr}(\bar{D}^2V)(D^2V)$$
$$+ \int \mathrm{d}^4y\, \mathrm{d}^2\theta \frac{\mathrm{i}}{2}\,\mathrm{Tr}\, b\bar{D}^2\delta_{\bar{c},c}V + \int \mathrm{d}^4\bar{y}\, \mathrm{d}^2\bar{\theta} \frac{\mathrm{i}}{2}\,\mathrm{Tr}\, \bar{b}D^2\delta_{\bar{c},c}V. \tag{A6}$$

Below we concentrate on the gauge part of the action. A short review of the procedure necessary to remove divergences from the effective action is given. This review is necessary to compare with the case of softly broken supersymmetry analysed in the main part of this paper. This review is very concise and anybody who is interested in more details can refer to the reviews [10, 11]. The BRST symmetry is reviewed in [11] and applications of Slavnov–Taylor identities to the renormalization of supersymmetric theories can be found in [10].

The action (A6) is invariant under the BRST symmetry,

$$\begin{aligned} &e^V \to e^{i\bar{c}\varepsilon} e^V e^{ic\varepsilon} \qquad \delta b = \frac{1}{32}\frac{1}{\alpha}(\bar{D}^2 D^2 V)\varepsilon \\ &c \to c + ic^2\varepsilon \qquad \delta \bar{b} = \frac{1}{32}\frac{1}{\alpha}(D^2 \bar{D}^2 V)\varepsilon \\ &\bar{c} \to \bar{c} - i\bar{c}^2\varepsilon \end{aligned} \tag{A7}$$

with a Hermitian–Grassmannian parameter ε, $\varepsilon^\dagger = \varepsilon$. This looks like a gauge transformation for the vector superfield (A4). The transformation of the ghost superfields is caused by the transformation of $\delta_{\bar{c},c}V$ under the BRST transformation of V in (A7). By construction, $\delta_{\bar{c},c}V$ is the solution to the equation (A5) when $\bar{\Lambda}$, Λ are replaced with $\bar{c}$, c respectively. If in the equation (A5) we put the transformed vector superfield $V + \delta_{i\bar{c}\varepsilon, ic\varepsilon}V$ according to

$$e^{V+\delta_{i\bar{c}\varepsilon, ic\varepsilon}V} = e^{i\bar{c}\varepsilon} e^V e^{ic\varepsilon}$$

instead of V, we obtain that the solution $\delta_{\bar{c},c}V$ to equation (A5) takes the transformation $\delta(\delta_{\bar{c},c}V)$ that satisfies the equation

$$e^V(\delta(\delta_{\bar{c},c}V)) - (\delta(\delta_{\bar{c},c}V))e^V = [V, i\bar{c}^2\varepsilon]e^V + e^V[V, -ic^2\varepsilon].$$

The transformations of the ghost superfields in (A7) compensate this transformation of $\delta_{\bar{c},c}V$, so that the total BRST transformation of $\delta_{\bar{c},c}V$ is vanishing,

$$\delta_{\mathrm{BRST}}(\delta_{\bar{c},c}V) = 0.$$

At the same time, the transformation of antighost superfields b, $\bar{b}$ is necessary to remove the non-invariance of the gauge fixing term.

The path integral for the rigid theory is defined as

$$\begin{aligned} Z[J, \eta, \bar{\eta}, \rho, \bar{\rho}, K, L, \bar{L}] = \int &\mathrm{d}V\, \mathrm{d}c\, \mathrm{d}\bar{c}\, \mathrm{d}b\, \mathrm{d}\bar{b} \exp i[S^{\mathrm{R}}_{\mathrm{gauge}} \\ &+2\ \mathrm{Tr}(JV + i\eta c + i\bar{\eta}\bar{c} + i\rho b + i\bar{\rho}\bar{b}) + 2\,\mathrm{Tr}(iK\delta_{\bar{c},c}V + Lc^2 + \bar{L}\bar{c}^2)]. \end{aligned} \tag{A8}$$

The third term in the brackets is the BRST invariant since the external superfields K and L are BRST invariant by definition. All fields in the path integral are in the adjoint representation of the gauge group. For the sake of brevity we omit the symbol of integration in the terms with external sources, keeping in mind that it is the full superspace measure for vector superfields and the chiral measure for chiral superfields.

Having made the change of fields in the path integral

$$b \to b + \varepsilon \qquad \bar{b} \to \bar{b} + \bar{\varepsilon}$$

with an arbitrary chiral superfield ε, two identities can be obtained:

$$\bar{\rho} - i\frac{1}{4}D^2\frac{\delta W}{\delta K} = 0 \qquad \rho - i\frac{1}{4}\bar{D}^2\frac{\delta W}{\delta K} = 0 \tag{A9}$$

where the standard definition for the connected diagram generator is used,

$$Z = e^{-iW}.$$

For the derivative with respect to the vector superfield we use the definition

$$\frac{\delta}{\delta K} \equiv T^a \frac{\delta}{\delta K^a}$$

while the derivative with respect to the chiral superfield is defined from the requirement

$$\frac{\delta}{\delta \eta(y,\theta)} \int \mathrm{d}^4 y'\, \mathrm{d}^2\theta'\, 2\,\mathrm{Tr}\, \eta(y',\theta') c(y',\theta') = c(y,\theta) \Rightarrow \frac{\delta \eta^a(y',\theta')}{\delta \eta^b(y,\theta)} = \tfrac{1}{4}\bar{D}^2 \delta^{(8)}(z-z')\delta^{ab}.$$

Here z is the definition for the total superspace coordinate $z = (x, \theta, \bar{\theta})$, so

$$\delta^{(8)}(z-z') = \delta^{(4)}(x-x')\delta^{(2)}(\theta-\theta')\delta^{(2)}(\bar{\theta}-\bar{\theta}').$$

The effective action Γ is related to W by the Legendre transformation

$$V \equiv -\frac{\delta W}{\delta J} \qquad \mathrm{i}c \equiv -\frac{\delta W}{\delta \eta} \qquad \mathrm{i}\bar{c} \equiv -\frac{\delta W}{\delta \bar{\eta}} \qquad \mathrm{i}b \equiv -\frac{\delta W}{\delta \rho} \qquad \mathrm{i}\bar{b} \equiv -\frac{\delta W}{\delta \bar{\rho}} \tag{A10}$$

$$\begin{aligned} &\Gamma = -W - 2\,\mathrm{Tr}(JV + \mathrm{i}\eta c + \mathrm{i}\bar{\eta}\bar{c} + \mathrm{i}\rho b + \mathrm{i}\bar{\rho}\bar{b}) \equiv -W - 2\,\mathrm{Tr}(X\phi) \\ &(X\phi) \equiv \mathrm{i}^{G(k)} X^k \phi^k \\ &X \equiv (J, \eta, \bar{\eta}, \rho, \bar{\rho}) \qquad \phi \equiv (V, c, \bar{c}, b, \bar{b}) \end{aligned} \tag{A11}$$

where $G(k) = 0$ if ϕ^k is the Bose superfield and $G(k) = 1$ if ϕ^k is the Fermi superfield. Iteratively all equations (A10) can be reversed,

$$X = X[\phi, K, L, \bar{L}]$$

and the effective action is defined in terms of new variables, $\Gamma = \Gamma[\phi, K, L, \bar{L}]$. Hence, the following equalities occur:

$$\begin{aligned} &\frac{\delta \Gamma}{\delta V} = -\frac{\delta X^a}{\delta V}\frac{\delta W}{\delta X^a} - \mathrm{i}^{G(a)}\frac{\delta X^a}{\delta V}\phi^a - J = -J \\ &\frac{\delta \Gamma}{\delta K} = -\frac{\delta X^a}{\delta K}\frac{\delta W}{\delta X^a} - \mathrm{i}^{G(a)}\frac{\delta X^a}{\delta K}\phi^a - \frac{\delta W}{\delta K} = -\frac{\delta W}{\delta K} \\ &\frac{\delta \Gamma}{\delta c} = \mathrm{i}\eta \qquad \frac{\delta \Gamma}{\delta \bar{c}} = \mathrm{i}\bar{\eta} \qquad \frac{\delta \Gamma}{\delta b} = \mathrm{i}\rho \qquad \frac{\delta \Gamma}{\delta \bar{b}} = \mathrm{i}\bar{\rho} \\ &\frac{\delta \Gamma}{\delta L} = -\frac{\delta W}{\delta L} \qquad \frac{\delta \Gamma}{\delta \bar{L}} = -\frac{\delta W}{\delta \bar{L}}. \end{aligned} \tag{A12}$$

Here all Grassmannian derivatives are left derivatives. Therefore, the ghost equations (A9) can be written as

$$\frac{\delta \Gamma}{\delta \bar{b}} - \frac{1}{4}D^2 \frac{\delta \Gamma}{\delta K} = 0 \qquad \frac{\delta \Gamma}{\delta b} - \frac{1}{4}\bar{D}^2 \frac{\delta \Gamma}{\delta K} = 0. \tag{A13}$$

If the change of fields (A7) in the path integral (A8) is made, that we obtain the Slavnov–Taylor identity as the result of invariance of the integral (A8) under a change of variables,

$$\mathrm{Tr}\left[J\frac{\delta}{\delta K} - \mathrm{i}\eta\left(\frac{1}{\mathrm{i}}\frac{\delta}{\delta L}\right) + \mathrm{i}\bar{\eta}\left(\frac{1}{\mathrm{i}}\frac{\delta}{\delta \bar{L}}\right) + \mathrm{i}\rho\left(\frac{1}{32}\frac{1}{\alpha}\bar{D}^2 D^2 \frac{\delta}{\delta J}\right) + \mathrm{i}\bar{\rho}\left(\frac{1}{32}\frac{1}{\alpha}D^2 \bar{D}^2 \frac{\delta}{\delta J}\right)\right] W = 0$$

or, taking into account the relations (A12), we have

$$\mathrm{Tr}\left[\frac{\delta \Gamma}{\delta V}\frac{\delta \Gamma}{\delta K} - \mathrm{i}\frac{\delta \Gamma}{\delta c}\frac{\delta \Gamma}{\delta L} + \mathrm{i}\frac{\delta \Gamma}{\delta \bar{c}}\frac{\delta \Gamma}{\delta \bar{L}} - \frac{\delta \Gamma}{\delta b}\left(\frac{1}{32}\frac{1}{\alpha}\bar{D}^2 D^2 V\right) - \frac{\delta \Gamma}{\delta \bar{b}}\left(\frac{1}{32}\frac{1}{\alpha}D^2 \bar{D}^2 V\right)\right] = 0. \tag{A14}$$

The identities (A13) and (A14) allow us to remove all possible divergences from the effective action Γ by rescaling superfields and couplings in the classical action (A6). Indeed, the identity (A13) restricts the dependence of Γ on the antighost superfields and on the external source K to an arbitrary dependence on their combination $b + \bar{b} + K$. This means that the corresponding singular part of the effective action is

$$\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, 2\mathrm{i}\, \mathrm{Tr}(b + \bar{b} + K)A(x, \theta, \bar{\theta})$$

where $A(x, \theta, \bar{\theta})$ is a combination of $c, \bar{c}, V$. By index counting arguments we know that the singular part repeats the structure of the classical action (A6) up to coefficients. Hence, $A(x, \theta, \bar{\theta})$ starts from the $z_1(c+\bar{c})$, since Γ is Hermitian. Here z_1 is a constant that can be found by using the supergraph technique. The Slavnov–Taylor identity (A14) fixes the coefficient before the longitudinal part of the two-point vector Green function. Indeed, by using projectors from (A1) the two-point vector correlator can be decomposed into the sum of transversal and longitudinal parts. The identity (A14) means that there is neither infinite nor finite correction to the longitudinal part of the two-point vector correlator

$$\frac{1}{\alpha}\frac{1}{32}V(D^2\bar{D}^2 + \bar{D}^2D^2)V$$

even if we have taken into account the whole dependence of the effective action Γ on the combination $b + \bar{b} + K$.

Now it is necessary to consider contributions to $A(x, \theta, \bar{\theta})$ of the next orders in fields. For example, the third-order terms can be presented as

$$\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, 2\mathrm{i}\, \mathrm{Tr}(b + \bar{b} + K)[z_1(c + \bar{c}) + z_4(Vc + \bar{c}V) + z_5(cV + V\bar{c})] \\ + \int \mathrm{d}^4y\, \mathrm{d}^2\theta\, 2\,\mathrm{Tr}\, z_6Lc^2 + \int \mathrm{d}^4\bar{y}\, \mathrm{d}^2\bar{\theta}\, 2\,\mathrm{Tr}\, \bar{z}_6\bar{L}\bar{c}^2. \tag{A15}$$

By the no-renormalization theorem for the superpotential [9] we obtain

$$z_6 = \bar{z}_6 = 1.$$

To fix the constants z_4 and z_5, we make the change of variables in the effective action Γ,

$$\Gamma[V, c, \bar{c}, b, \bar{b}, K, L, \bar{L}] = \Gamma[V(\tilde{V}), c, \bar{c}, b, \bar{b}, K(\tilde{K}), L, \bar{L}] = \tilde{\Gamma}[\tilde{V}, c, \bar{c}, b, \bar{b}, \tilde{K}, L, \bar{L}] \\ V = \tilde{V}z_1 \qquad K = \frac{\tilde{K}}{z_1}. \tag{A16}$$

The Slavnov–Taylor identity (A14) in new variables is

$$\mathrm{Tr}\left[\frac{\delta\tilde{\Gamma}}{\delta\tilde{V}}\frac{\delta\tilde{\Gamma}}{\delta\tilde{K}} - \mathrm{i}\frac{\delta\tilde{\Gamma}}{\delta c}\frac{\delta\tilde{\Gamma}}{\delta L} + \mathrm{i}\frac{\delta\tilde{\Gamma}}{\delta\bar{c}}\frac{\delta\tilde{\Gamma}}{\delta\bar{L}} - \frac{\delta\tilde{\Gamma}}{\delta b}\left(\frac{1}{32}\frac{1}{\alpha}\bar{D}^2D^2\tilde{V}z_1\right) - \frac{\delta\tilde{\Gamma}}{\delta\bar{b}}\left(\frac{1}{32}\frac{1}{\alpha}D^2\bar{D}^2\tilde{V}z_1\right)\right] = 0. \tag{A17}$$

The part of the effective action (A15) in the new variables looks like

$$\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, 2\mathrm{i}\, \mathrm{Tr}((b + \bar{b})z_1 + \tilde{K})[(c + \bar{c}) + z_4'(\tilde{V}c + \bar{c}\tilde{V}) + z_5'(c\tilde{V} + \tilde{V}\bar{c})] \\ + \int \mathrm{d}^4y\, \mathrm{d}^2\theta\, 2\,\mathrm{Tr}\, Lc^2 + \int \mathrm{d}^4\bar{y}\, \mathrm{d}^2\bar{\theta}\, 2\,\mathrm{Tr}\, \bar{L}\bar{c}^2 \tag{A18}$$

where z_4' and z_5' are new constants.

The higher-order terms in the brackets of (A18) are restored unambiguously by themselves in an iterative way due to the first three terms in the modified identities (A17). As the result we have

$$\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, 2\mathrm{i}\, \mathrm{Tr}((b + \bar{b})z_1 + \tilde{K})[\delta_{\bar{c},c}\tilde{V}]. \tag{A19}$$

Now it is necessary to consider the transversal part of the two-point vector correlator. Having made the change of variables (A16) in the effective action, we obtain the transversal part as

$$\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, z_3 z_1^2 \frac{1}{g^2}\frac{1}{2^5}\,\mathrm{Tr}\, D_\alpha \tilde{V} \bar{D}^2 D^\alpha \tilde{V} + \mathrm{H.c.}. \tag{A20}$$

This is the only gauge invariant combination fixed by the first term in the modified identities (A17), if we take into account the already fixed dependence (A19) of the singular part of $\tilde{\Gamma}$ on the external source $\tilde{K}$. Here z_3 is a constant that can be found by using the supergraph technique [12].

The first term in the modified identity (A17) will restore in the iterative way higher-order terms starting from the bilinear transversal two-point correlator (A20). Hence, the result of this restoration is

$$\int \mathrm{d}^4y\, \mathrm{d}^2\theta\, \frac{1}{g^2}\frac{1}{2^7} z_3 z_1^2\, \mathrm{Tr}\, W_\alpha(\tilde{V}) W^\alpha(\tilde{V}) + \mathrm{H.c.}. \tag{A21}$$

The singular part of the effective action $\tilde{\Gamma}$ can be written as a combination of (A21) and (A19),

$$\begin{aligned}\tilde{\Gamma}_{\mathrm{sing}} = &\int \mathrm{d}^4y\, \mathrm{d}^2\theta\, \frac{1}{g^2}\frac{1}{2^7} z_3 z_1^2\, \mathrm{Tr}\, W_\alpha(\tilde{V}) W^\alpha(\tilde{V}) + \mathrm{H.c.}\\ &+\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \mathrm{Tr}\, \frac{1}{\alpha}\frac{1}{32} z_1 \tilde{V}(D^2\bar{D}^2 + \bar{D}^2 D^2) z_1 \tilde{V}\\ &+\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, 2\mathrm{i}\, \mathrm{Tr}((b+\bar{b})z_1 + \tilde{K})[\delta_{\bar{c},c}\tilde{V}].\end{aligned}$$

Now we should go back to the initial variables V and K, that is, we should make the change of variables in $\tilde{\Gamma}$ opposite to (A16). Hence, the singular part of the effective action which corresponds to the theory with the classical action (A6) is

$$\begin{aligned}\Gamma_{\mathrm{sing}} = &\int \mathrm{d}^4y\, \mathrm{d}^2\theta\, \frac{1}{g^2}\frac{1}{2^7} z_3 z_1^2\, \mathrm{Tr}\, W_\alpha\left(\frac{V}{z_1}\right) W^\alpha\left(\frac{V}{z_1}\right) + \mathrm{H.c.}\\ &+\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \mathrm{Tr}\, \frac{1}{\alpha}\frac{1}{32} V(D^2\bar{D}^2 + \bar{D}^2 D^2) V\\ &+\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, 2\mathrm{i}\, \mathrm{Tr}((b+\bar{b})z_1 + K z_1)\left[\delta_{\bar{c},c}\left(\frac{V}{z_1}\right)\right].\end{aligned} \tag{A22}$$

Hence, all possible divergences can be removed from Γ_{sing} by the following rescaling of fields and couplings in the path integral (A8):

$$V = V_R\, z_1 \qquad \frac{1}{g^2} = \frac{1}{g_R^2} z_1^{-2} z_3^{-1} \qquad \alpha = z_1^2\, \alpha_R \qquad b = b_R\, z_1^{-1} \qquad K = K_R\, z_1^{-1}.$$

References

[1] Yamada Y 1994 *Phys. Rev.* D **50** 3537
[2] Hisano J and Shifman M 1997 *Phys. Rev.* D **56** 5475
[3] Giudice G F and Rattazzi R 1998 *Nucl. Phys.* B **511** 25
[4] Jack I and Jones D R T 1997 *Phys. Lett.* B **415** 383
[5] Avdeev L V, Kazakov D I and Kondrashuk I N 1998 *Nucl. Phys.* B **510** 289
[6] Arkani-Hamed N, Giudice G F, Luty M A and Rattazzi R 1998 *Phys. Rev.* D **58** 115005
[7] Kondrashuk I 1999 *Phys. Lett.* B **470** 129
[8] Luty M A and Rattazzi R 1999 *J. High Energy Phys.* JHEP11(1999)001

[9] West P 1986 *Introduction to Supersymmetry and Supergravity* (Singapore: World Scientific)
[10] Piguet O 1996 Supersymmetry, supercurrent, and scale invariance *Preprint* hep-th/9611003
[11] Becchi C 1996 Introduction to BRS symmetry *Preprint* hep-th/9607181
[12] Grisaru M T, Siegel W and Roček M 1979 *Nucl. Phys.* B **159** 429
[13] Dine M and Nelson A E 1993 *Phys. Rev.* D **48** 1277
Dine M, Nelson A E and Shirman Yu 1995 *Phys. Rev.* D **51** 1362
Dine M, Nelson A E, Nir Y and Shirman Yu 1996 *Phys. Rev.* D **53** 2658
Giudice G F and Rattazzi R 1999 *Phys. Rep.* **322** 419
(Giudice G F and Rattazzi R 1998 *Preprint* hep-ph/9801271)
[14] Slavnov A A 1998 *Theor. Math. Phys.* **117** 1364
[15] Kobayashi T, Kubo J and Zoupanos G 1998 *Phys. Lett.* B **427** 291
Kribs G D 1998 *Nucl. Phys.* B **535** 41
Kobayashi T, Kubo J, Mondragon M and Zoupanos G 1999 *Nucl. Phys.* B **550** 99
Huitu K and Kobayashi T 1999 *Phys. Lett.* B **470** 901
Jack I and Jones D R T 1999 *Phys. Lett.* B **465** 148
Kazakov D and Moultaka G 2000 *Nucl. Phys.* B **577** 121
Kazakov D I 2000 Supersymmetry in particle physics: the renormalization group viewpoint *Preprint* hep-ph/0001257
[16] Siegel W 1979 *Phys. Lett.* B **84** 193
Capper D M, Jones D R T and van Nieuwenhuizen P 1980 *Nucl. Phys.* B **167** 479
Gates J Jr, Grisaru M T, Roček M and Siegel W 1983 *One Thousand and One Lessons in Supersymmetry* (New York: Benjamin-Cummings)
[17] Grisaru M T, Milewski B and Zanon D 1985 *Phys. Lett.* B **155** 357
[18] Jack I, Jones D R T, Martin S P, Vaughn M T and Yamada Y 1994 *Phys. Rev.* D **50** 5481
[19] Jack I, Jones D R T and Pickering A 1998 *Phys. Lett.* B **426** 73
[20] Jack I, Jones D R T and Pickering A 1998 *Phys. Lett.* B **432** 114
[21] Jack I and Jones D R T 1999 *Phys. Lett.* B **465** 148